Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED. A. M., (Princeton), Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the May Number.]

PROPOSITION XIX. *Let there be any triangle AHD* (fig.18) *right angled at H. Then in AD produced the portion DC is assumed equal to this AD; and the perpendicular CB is let fall to AH produced. I say hence will be established the hypothesis of right angle, or obtuse angle, or acute angle, according as the portion HB is equal to, or greater, or less than this AH.*

PROOF. For the join DB will be (Eu.I.4, and P. X of this) either equal to, or greater, or less than AD, or DC, according as the portion HB is equal to, or greater, or less than AH. And first indeed let HB be equal to AH, so that therefore the join DB may be equal to AD, or DC.

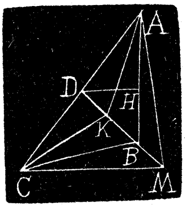

Fig. 18.

It follows that the circumference of the circle, which is described with the center D, and radius DB, will go through the points A, and C. Therefore the angle ABC, which is assumed right, is in this semicircle, whose diameter is AC. Wherefore (from the preceding proposition) is established the hypothesis of right angle. Quod erat primo loco demonstrandum.

Secondly let BH be greater than AH, so that therefore the join DB is greater than AD, or DC. It follows that the circumference of the circle, which is described with center D, and radius DA, or DC, will meet DB in some intermediate point K. Therefore, AK, and CK being joined, the angle AKC will be obtuse, because greater (Eu. I. 21) than the angle ABC, which is assumed right. Wherefore (from the preceding proposition) is established the hypothesis of obtuse angle. Quod erat secundo loco demonstrandum.

Thirdly let BH be less than AH, so that therefore the join DB is less than AD, or DC. It follows that the circumference of the circle, which is described with center D, and radius DA, or DC, will meet in some point M this DB produced outwardly. Therefore AM, and CM being joined, the angle AMC will be acute, because less (Eu. I. 21) than the angle ABC, which is assumed right.

Therefore (from the preceding proposition) is established the hypothesis of acute angle. Quod erat tertio loco demonstrandum. Itaque constant omnia proposito.

[To be continued.]